CMSC 426 Midterm Exam (Questions) Spring 2019

1. Consider the following 5×5 pixel image.

0	0	0	0	0
0	0	0	0	6
0	0	0	6	6
0	0	6	6	6
0	6	6	6	6

(a) Cross correlate with the following two 3×3 filters, which approximate the x-derivative and y-derivative, respectively. Compute the output image. You may start cross correlation by centering the filter at pixel (2,2) of the image and end at pixel (4,4) of the image. So your output images will be of size 3×3

$$\frac{1}{6}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad \frac{1}{6}\begin{array}{|c|c|c|}\hline -1 & -1 & -1 \\ \hline 0 & 0 & 0 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

0 1 2
1 2 2
2 2 1

0 1 2
1 2 2
2 2 1

(b) What is the edge strength and direction at the center pixel ((3,3) in the image)?

θ: $\operatorname{atan} \frac{2}{2} = 45^\circ$

abs: $\sqrt{2^2+2^2} = \sqrt{2} \cdot 2$

2. The SIFT algorithm for detection of interest points requires convolution of an image with Gaussians of different scales within an octave. Answer the following questions:

   (a) If within an octave the scales of kernels differ by a constant factor k. Starting with a scale of σ_0 for the first kernel, what would be the scale of the next four Gaussian kernels?

   (b) If there are $n+1$ images of different scale in an octave. What is the scale factor, k, in terms of n.

   (c) What should be the standard deviation (σ) of the Gaussian kernel so that we would be able to convolve only once and still get the same output as convolving twice with a kernel of standard deviation, σ.

(a) $\sigma_0 K \quad \sigma_0 K^2 \quad \sigma_0 K^3 \quad \sigma_0 K^4$

(b) $\sqrt[n]{2}$

(c) $\sqrt{2}\,\sigma$

3. Write in pseudo code a program that takes as input an edge image (with gradient orientation) and finds the center of circles of radius R= 5 pixels using the Hough Transform. Your program should take the gradient orientation into account and vote in 2-D space for the coordinates (x_0, y_0) of the circle center.

datastructure that holds edges edges $(\cancel{x}, y, \theta)$

1) Initialize $H[x, y] = 0$

2 for each edge point $e(x, y, \theta)$

$(x_0, y_0) = \text{round}(x + 5\cos\theta, \cancel{x} + 5\sin\theta)$

$H[x_0, y_0] += 1$

$(x_0, y_0) = \text{round}(x - 5\cos\theta, y - 5\sin\theta)$

$H[x_0, y_0] += 1$

end for

3 Find values $[x, y]$ where $H[x, y] >$ threshold

4. A planar object with points $A = (1, 0)$ and $B = (3, 0)$ is transformed with a similarity transform so that the new points are $A' = (\sqrt{2}+1, \sqrt{2}+1)$ and $B' = (3\sqrt{2}+1, 3\sqrt{2}+1)$.

   (a) Compute the four parameters θ, s, t_x, t_y of the similarity transform.
   Hint: In a similarity transform points are related as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta \\ s\sin\theta & s\cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

   (b) Instead you are given three noisy point correspondencs of the same similarity transform A = (x_A, y_A), B = (x_B, y_B), C= (x_C, y_C), A' = (x'_A, y'_A), B' = (x'_B, y'_B), C'= (x'_C, y'_C). Explain how you will solve for the four parameters of the similarity transform.

(a) $s\cos\theta x - s\sin\theta y + t_x = x'$

$s\sin\theta x + s\cos\theta y + t_y = y'$

A I $s\cos\theta \quad - \quad + t_x = \sqrt{2}+1$

II $s\sin\theta \quad + t_y = \sqrt{2}+1$

B III $3s\cos\theta \quad + t_x = 3\sqrt{2}+1$

IV $3s\sin\theta \quad + t_y = 3\sqrt{2}+1$

III $-3\cdot$I $\quad 2t_x = 2 \rightarrow \boxed{t_x = 1}$

IV -3II $\quad 2t_y = 2 \quad \boxed{t_y = 1}$

II/I $\quad \tan\theta = 1 \rightarrow \boxed{\theta = 45°}$

I $\quad s\frac{1}{2}\sqrt{2} = \sqrt{2} \rightarrow \boxed{s = 2}$

(b) $s\cos\theta = a \qquad +s\sin\theta = b$

$x' = ax - by + t_x$

$y' = bx + ay + t_y \rightarrow$

$$\begin{bmatrix} x_A & -y_A & 1 & 0 \\ y_A & x_A & 0 & 1 \\ x_B & -y_B & 1 & 0 \\ y_B & x_B & 0 & 1 \\ x_C & -y_C & 1 & 0 \\ y_C & x_C & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ t_x \\ t_y \end{bmatrix}$$

$a, b \rightarrow \quad \theta = \text{atan}\frac{b}{Y} \qquad s = \sqrt{a^2+b^2}$

5. Assume the following scene shown below with a background plane at 3m distance and three objects (a rectangle, a circle and a triangle) at 1m distance, as drawn below. Draw for the following three motions qualitatively the motion vectors in the three subfigures of Figure 1.

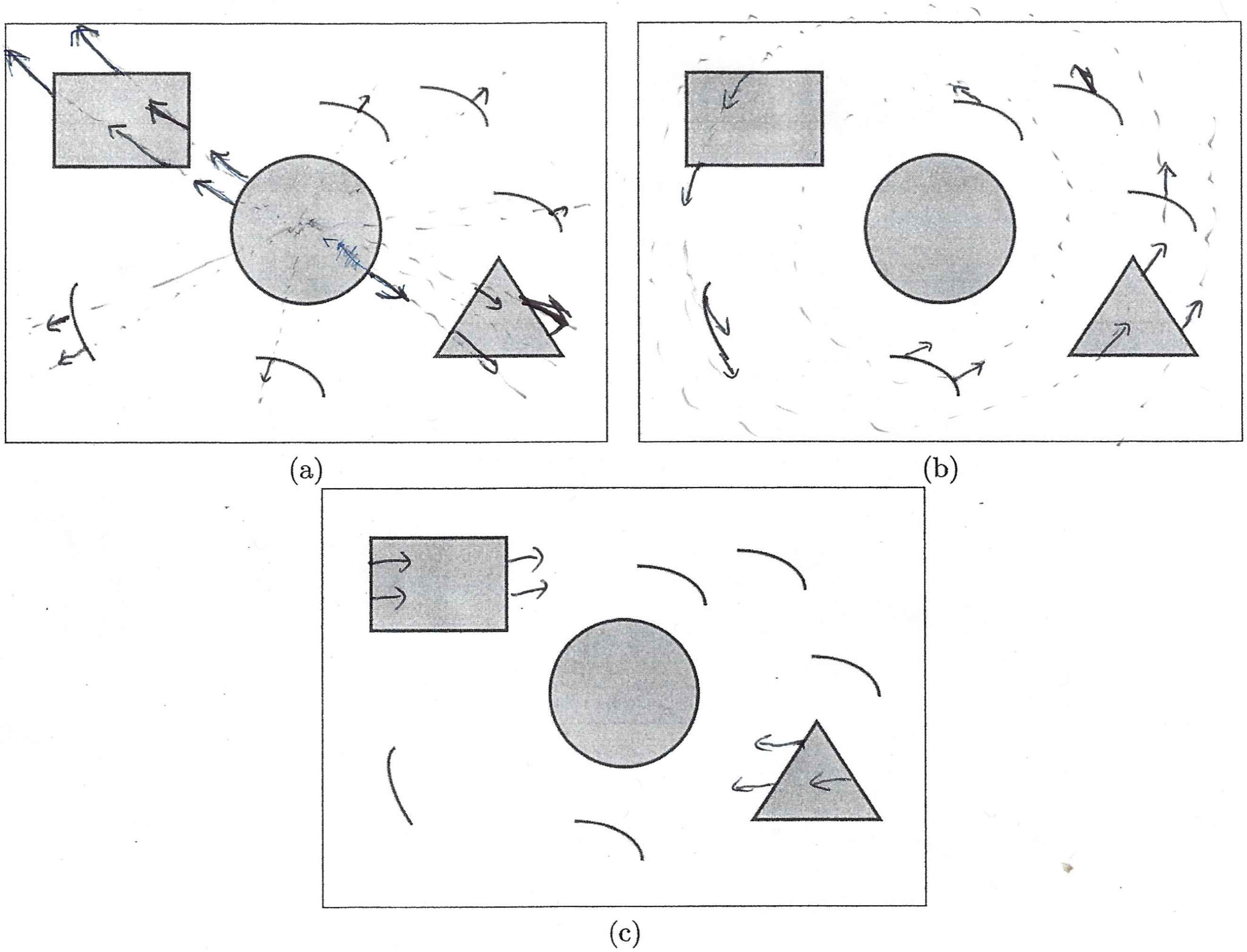

Figure 1: Two cameras view a world plane.

(a) The camera is moving towards the scene with a translation parallel to the optical axis (or Z-axis).

(b) The camera is rotating clockwise around the optical axis (or Z-axis).

(c) The rectangle is moving with velocity 1m/sec to the right and the triangle is moving with velocity 1m/sec to the left.